Available online at: www.basra-science-journal.org

ISSN –1817 –2695

On the stability of an isentropic charged superdense star model

Dhuha Majeed Al-Yassiri[1] and Mahmood K. Jasim[2]

[1]Department of Mathematics, University of Suleimania, Suleimania, Iraq

Email: dhuha_alyassiri@yahoo.com

[2]Department of Mathematics & physical Sciences, University of Nizwa, Sultanate of Oman

Email: mahmoodkhalid@unizwa.edu.om

Received 30-10-2012 , Accepted 28-4-2013

Abstract

The objective of this paper is to study the stability of charged isentropic superdense star models. A limitation of the density variation for different models guided by a specific choice of a measure of departure from physical geometry of the physical space to ensure the physical acceptability have been obtained and analyzed. The solution so obtained has large density and pressure at the center, however the energy conditions are seen to be satisfied throughout certain spherical regions. In addition to that, these models are seen to satisfy various physical conditions. In our present paper the stability of charged superdense star has been found. Runge-Kutta method has been implemented for the superdense models which it has been found to satisfy various physical conditions.

Keywords: Einstein's field equations, Charged fluid spheres, Mathematical models.

1. Introduction

In general relativity and allied theories, the distribution of the mass, momentum, and stress due to matter and to any non-gravitational fields are described by the energy momentum tensor (or *matter tensor*) T^i_j .However, the Einstein field equation is not very choosy about what kinds of states of matter or non-gravitational fields are admissible in a space-time model [1]. For the last four decades research workers have been busy in deriving the solutions for charged fluid spheres to provide source of Reissener (1916) and Nordstrom (1918) solutions[10]. Such fluid models are not likely to undergo gravitational collapse to reduce into a point singularity, in the presence of charges. The gravitational

attraction may be nullified by electrostatic repulsion and pressure gradient [3].
Several workers have studied the number of charged fluids in different contexts such as Bonner 1960 Tikekar (1990),Felice 1995,Gupta et al (1986) ,Ray et al (2003),Moodely et al (2003), have found a class of accelerating, expanding and shearing solutions which is characterized geometrically by conformal Killing vector[6] while Thirukanesh and Maharaj found a new class of exact solutions in closed form and have mentioned that a physical analysis indicated that the model may be used to describe a charge sphere[2]. Gupta ,and Sunil Kumar (2005,2010), have charged the Vaidya-Tikekar type solutions[8], then many researchers followed have charged Buchdahl's fluid spheres [6], while, Gupta and Kumar at 2011 have found a class of analogues of Durgopal and Floria superdense star and the members of this class are seen to satisfy the various physical conditions[9].

2. *Basic Field Equations*

In standard coordinates $x^i = (t, r, \theta, \phi)$, such that x^i denotes the four dimension coordinates and i=1,2,3,4, then we can choose the general line element for a charged fluid sphere model as follows

$$ds^2 = -e^{\lambda(r)}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta\, d\phi^2 + e^{\nu(r)}dt^2 \tag{1}$$

where

$$\lambda(r) = \ln\left(\left(1 - K\frac{r^2}{R^2}\right) \Big/ \left(1 - \frac{r^2}{R^2}\right) \right) \tag{1a}$$

And

$$\nu(r) = \ln\left(\left(1 - K\frac{r^2}{R^2}\right)(1-K) \right)^3 \tag{1b}$$

represent the metric potential functions that satisfying the Einstein –Maxwell equations

$$R^i_j - \frac{1}{2}R\delta^i_j = -8\pi\frac{G}{C^4}T^i_j \tag{2}$$

Such that R^i_j represents a Ricci tensor

Where C=1, G=1

And

$$T^i_j = M^i_j + E^i_j \quad , \tag{3}$$

In the interior, M^i_j can be described by an isentropic pressure P and the mass density ρ, to take the form:

$$M^i_j = (P+\rho)u_i u^i - P\delta^i_j \tag{4}$$

where

$$u^i = \left(0,0,0,e^{-\nu/2}\right) \tag{5}$$

While, E^i_j, the electromagnetic contribution to the stress energy tensor can be written as:

$$E^i_j = -\frac{1}{4\pi}\left(F^{im}F_{jm} - \frac{1}{4}\delta^i_j F_{mn}F^{mn} \right) \tag{6}$$

F_{ik}, being the skew symmetric electromagnetic field tensor satisfying the Maxwell equations

$$F_{ik,j} + F_{kj,i} + F_{ji,k} = 0 \tag{7}$$

$$\frac{\partial}{\partial x^k}\left(\sqrt{-g}F^{ik}\right) = -4\pi\sqrt{-g}\, j^i \tag{8}$$

Where $j^i = \sigma v^i$ represents the four-current vector of charged fluid with σ as the charged density.

In view of (1) and (2) with (3), the field equation can be furnished as:

$$8\pi\rho = \frac{\lambda'}{r}e^{-\lambda} + \frac{\left(1-e^{-\lambda}\right)}{r^2} - 8\pi\left(E^i_j\right)^2 \tag{9}$$

$$8\pi P = \frac{\nu'}{r}e^{-\lambda} - \frac{\left(1-e^{-\lambda}\right)}{r^2} + 8\pi\left(E^i_j\right)^2 \tag{10}$$

$$8\pi P = \left[\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} + \frac{\nu' - \lambda'}{2r}\right] e^{-\lambda} - 8\pi\left(E^i_j\right)^2 \tag{11}$$

Where, prime denotes the differentiation with respect to *r* and

$$E^i_j = 4\pi\int_0^r \sigma\, r^2 e^{\lambda/2} dr = r^2\sqrt{-F_{14}F^{14}} = r^2 F^{41} e^{(\lambda+\nu)/2} \tag{12}$$

Which represent the total charge contained within the sphere of radius *r*

We proposed a charged fluid distribution by considering the electric field intensity (Gupta, et al 2005)

$$\frac{q^2}{r^4} = \frac{K^2 r^2 \gamma^2}{2R^2\left(K + R^2\right)} \tag{13}$$

Where K & γ being constants,

The consistency of the field equations (9)-(11) using (1a) & (1b) yield the hyper geometric equation

$$(1-X^2)\frac{d^2y}{dX^2} + X\frac{dy}{dX} + \left(1 - K + K\gamma^2\right)y = 0 \tag{14}$$

Where

$$X = \sqrt{\frac{K}{K-1}}\sqrt{1 + \frac{Cr^2}{K}}$$

$K < 0$ or $K > 1$ and $y = \sqrt{e^{\nu}}$

So, (14) can be solved exactly for two cases:

Case I: Null charged by putting $\gamma = 0$ [4],[5]

Case II: for charged case, the case has been discussed by Athraa-Jasim (2004), later on by Mukesh 2005 and Gupta 2011,which they have produced and they have discussed some physical properties of such kind problem while in our search in the next section, we have built an algorithm for discussing charged and null charged problem with the point of view of its physical properties for different value of *K* using 4th order Runge-Kutta method.

3. Dynamical Stability Of Fluid Sphere

The basic method for examining whether a relativistic charged fluid sphere is stable with respect to infinitesimal radial adiabatic pulsations that have been developed by Chandrasekhar (1964).

Vaidya and Tikekar (1982) obtained their solution by demanding that the energy-momentum tensor was that of perfect fluid [7]. However, the same static solution is in fact obtained assuming only that the static energy-momentum tensor is given by:

$$T_i^j = (-p, -p, -p, \rho)$$

Now, when investigating stability one is studying a dynamical object and to describe its behavior one needs to know the full, i.e., non-static energy momentum tensor. We restrict our analysis to the case where the energy-momentum tensor is given by that of a perfect fluid, i.e.

$$T_j^i = (P+\rho)u_i u^i - P g_j^i$$

To perform the stability analysis for our superdense star model, we restrict our investigation to the case where the fluid is isentropic under static conditions. in fact the restriction also, has been done by Vaidya and Tikekar (1982), when they claim that the speed of sound is given by $\frac{dp}{d\rho}$ which represents the rate of pressure with respect to density. This will only be the case if the fluid is isentropic, i.e., if the entropy per baryon is constant everywhere, since we generally have:

$$v_{\text{sound}}^2 = \left(\frac{\partial p}{\partial \rho}\right)_s,$$

where s denotes the entropy per baryon. Since the fluid flow is isentropic for a perfect fluid, it is thus constant everywhere and always if it is constant for the static case. Our analysis is thus valid at absolute zero (white dwarfs, neutron stars) or a star in convective equilibrium (super massive star).

Following Barden, et al (1966) writing:

$$u = \xi(r) r^2 e^{-v/2}$$

For the line element in equation (1)

The pulsation equation was implemented by Chandrasekhar's (1964) and can be furnished as:

$$\sigma^2 \int_{\text{center}}^{\text{boundary}} e^{\frac{3\lambda+v}{2}} (p+\rho)\frac{u^2}{r^2}\, dr = \int_{\text{center}}^{\text{boundary}} e^{\left(\frac{3v+\lambda}{2}\right)\left(\frac{p+\rho}{r^2}\right)} *$$
$$\left\{\left[-\frac{2}{r}\frac{dv}{dr} - \frac{1}{4}\left(\frac{dv}{dr}\right)^2 + 8\pi p e^{\lambda}\right]u^2 + \frac{dp}{d\rho}\left(\frac{du}{dr}\right)^2\right\} dr$$

Conclusions

Besides the properties already discussed, we have the following necessary conditions for the model to be physically acceptable.

i. The pressure should be positive and decreasing through the star.
ii. The weak and strong energy condition should be satisfied.
iii. The adiabatic velocity of sound $\left(\frac{dp}{d\rho}\right)^{\frac{1}{2}}$ should be less than the velocity of light.
iv. The adiabatic index $\gamma = \frac{(\rho+p)}{p} \bullet \frac{dp}{d\rho}$ should be larger than unity for the

temperature away from the center, or even be larger than 4/3 to prevent instability under radial perturbations. The later condition is however only a necessary but not sufficient to obtain a dynamically stable model.

The following figures show the behavior of standard physical quantities inside the Star for the least admissible values of λ (is L in graphs) to ensure the physical properties.

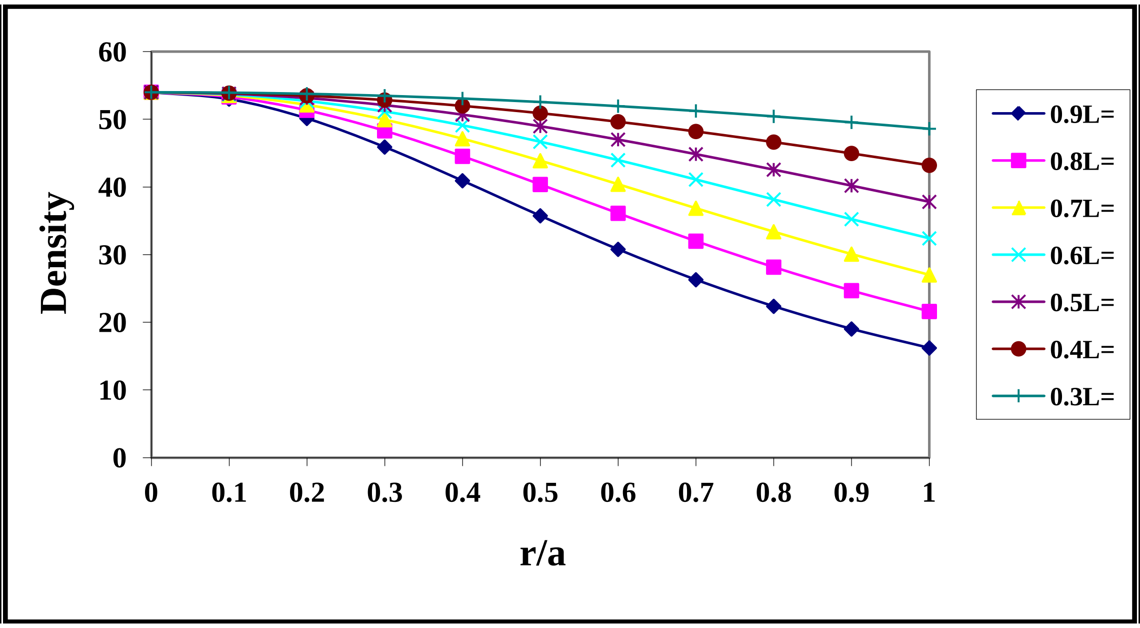

Fig.(1) Shows the Behavior of density inside the fluid sphere at (K = –11).

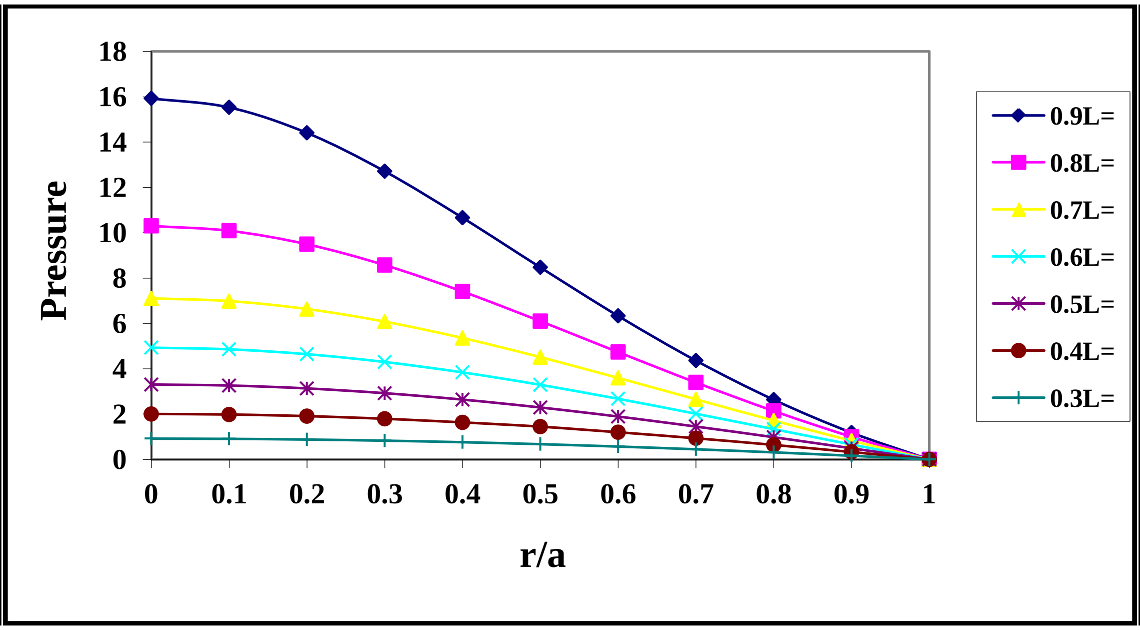

Fig.(2) Shows the Behavior of pressure inside the fluid sphere at (K = –11).

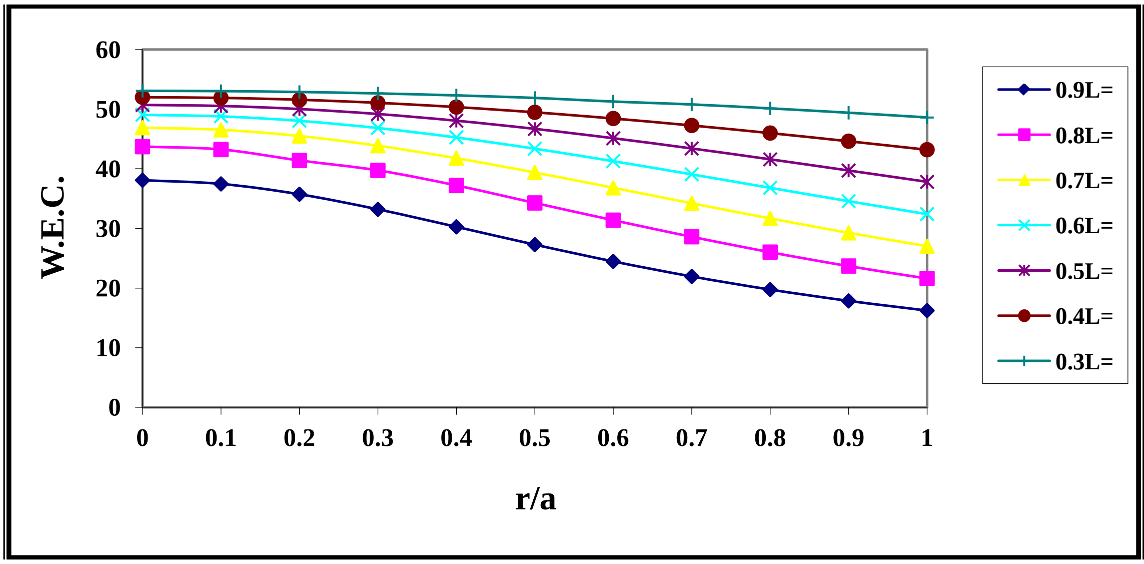

Fig.(3) Shows the Behavior of W.E.C. inside the fluid sphere at (K=–11).

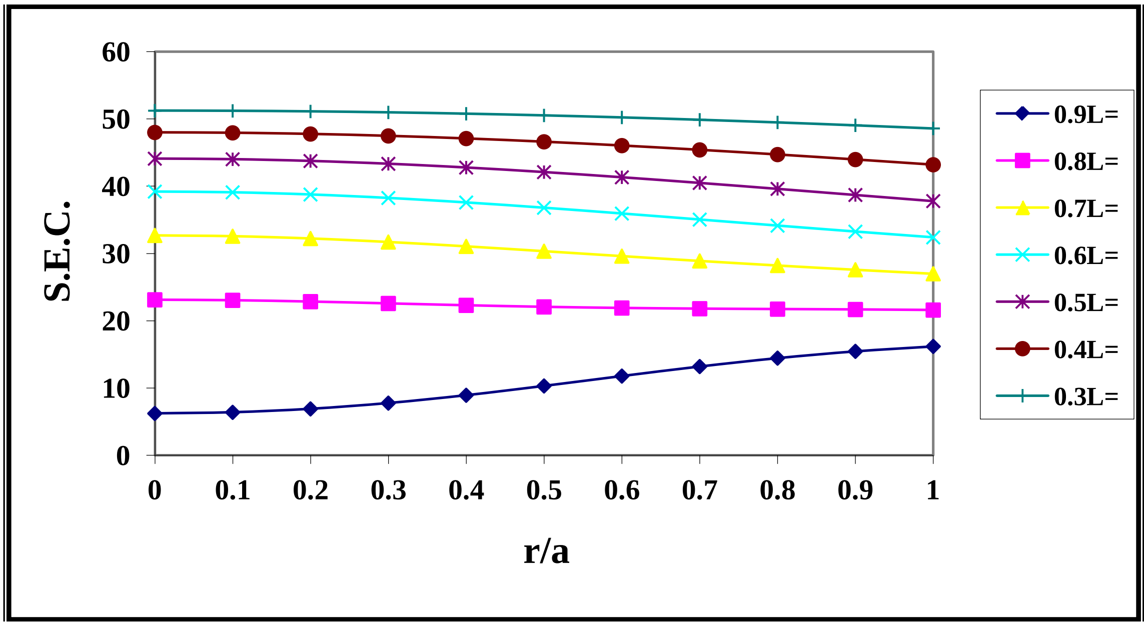

Fig..4) Shows the Behavior of S.E.C. inside the fluid sphere at (K=–11).

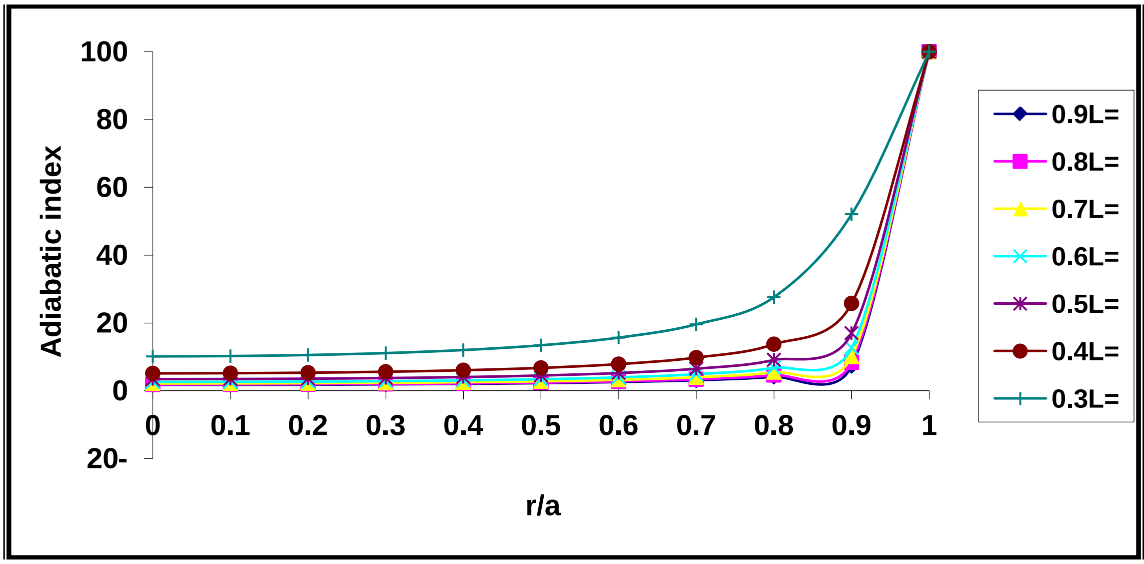

Fig.(5) Shows the Behavior of adiabatic index inside the fluid sphere at (K= −11).

References

[1] Buchdhal H. A., "General Relativistic Fluid Spheres", Phys. Rev., Vol.4, p.1027, 1959.

[2] Barden, James. M. Kips. Thorne and David Meltzer , "Catalogue of Method for Studying the Normal Mode of Radial Pulsation of General Relativistic Stellar Models" Astrophysics Journal ., No.2, Vol. 145, p 505, 1966.

[3] Knutsen H., "On the Stability and Physical Properties of an Exact Relativistic Model for A Superdense Star", Mon. Not. R. Soc., Vol.232, p.163-174, 1988.

[4] Knutsen, H., " Physical Properties of an Exact Spherically Symmetric Solution with Shear in General Relativity" GRG, Vol. 24 ,No.12, 1992

[5] Chandrasekhar, S., "The Dynamical Instability of Gaseous Mass Approaching the Schwarzschild Limit in General Relativity", Phys. Rev., Vol.140, No.2, P.417, 1964.

[6] Athraa A K , Jasim M K and Abul Samee Al-Janabi, "Some relativistic models of charged fluid spheres in terms of differential equations", Ph D thesis, Baghdad University, Iraq, 2004.

[7] Vaidya P. C. and Tikekar, R., "Exact Relativistic Model for a Superdense Star", Astrophysics & Astron, Vol.3, p.325, 1982

[8] Tikekar .R, "Exact model for a relativistic star", *J. Math. Phys.* **31** (1990), pp. 2454–2458

[9] Gupta, Y K and Mukesh Kumar, "On Charged analogues of Buchdahl's type fluid spheres, Astrophysics & space sciences" 299: 43-59, 2005.

[10] Gupta, Y K and Sunil Kumar, "A class of regular and well behaved charged analogues of Kuchowicz's relativistic super-dense star model", Astrophysics & space sciences 332: 415-421, 2011.

حول الأستقرارية لموديل من نجم مكتظ غيرموحد الخواص مشحون

كلزةخش

الهدف من هذا البحث لدراسة أستقرارية موديلات لنجم مكتظ غير موحد الخواص مشحون .أن تحديد تغيير الكثافة لموديلات مختلفة تقود الى أختيار محدد لقياس غياب الهندسة الفيزياوية من الفضاء الفيزياوي لضمان حصولنا على أنها مقبولة فيزياويا وقابلة للتحليل .الحل الذي حصلنا عليه له كثافة عالية وضغط في المركز. على أية حال شروط الطاقة رأينا أنها متحققة من خلال مناطق كروية مؤكدة.بالأضافة لهذا فأن هذه الموديلات يبدو بأنها تحقق شروط فيزياوية كثيرة.